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Stock Return Asymmetry: Beyond Skewness

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Abstract

In this article, we propose two asymmetry measures for stock returns. Unlike the popular skewness measure, our measures are based on the distribution function of the data rather than just the third central moment. We present empirical evidence that the greater upside asymmetries calculated using our new measures imply lower average returns in the cross section of stocks. In contrast, when using the skewness measure, the relationship between asymmetry and returns is inconclusive.

I. Introduction

Stock return skewness and its asset pricing implications have received substantial attention in investment theory and practice. Arditti (1971), Zhang (2005), Kumar (2009), and Boyer, Mitton, and Vorkink (2010), among others, find that higher skewness is associated with a lower expected return. In addition,

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Conrad, Dittmar, and Ghysels (2013) and Amaya, Christoffersen, Jacobs, and Vasquez (2015) document similar findings for stocks with options available and stocks with intraday data available, respectively. In contrast, Xing, Zhang, and Zhao (2010) uncover a positive correlation between skewness and expected returns. Bali, Cakici, and Whitelaw (2011) further show that skewness is not statistically significant in explaining the cross-sectional variation in expected returns over the period from July 1962 to Dec. 2005. Overall, evidence on the impact of skewness on the cross-section of stock returns is mixed and inconclusive (see Bali, Engle, and Murray (2016), for an excellent survey). Theoretically, Tversky and Kahneman (1992), Barberis and Huang (2008), and Han, Hirshleifer, and Walden (2018) emphasize the importance of asymmetry for expected returns, implying that greater upside asymmetry is associated with a lower expected return. However, the question of how to measure return asymmetry empirically remains open.

In this article, we propose two new measures of asymmetry.[1] Our first measure is defined as the difference between the upside and downside return probabilities. The greater the measure, the greater the upside potential of the asset return in terms of probability. Our second measure is an entropy-scaled version of the first, which measures the absolute difference between the entire left-tail and right-tail densities more precisely.[2]

Our asymmetry measures serve two purposes. First, they can be used as tests for asymmetry in stock returns. Indeed, we find via simulations that our distribution-based asymmetry measures can detect asymmetry with greater power than a skewness test. In addition, we detect more distribution-based asymmetry when testing real data from the U.S. stock market. For example, for value-weighted size portfolios, the skewness test detects asymmetry only in the smallest decile, whereas our measures can reject the symmetry hypothesis for 4 deciles out of 10. We also apply our tests to individual stocks and find the percentage of asymmetry to be substantially higher than that shown by the skewness test (roughly 18% vs. 11%).

Second, our asymmetry measures explain the cross-section of stock returns better than skewness does. We show that, under certain conditions, our measures are equivalent to skewness. However, for a distribution with 0 skewness, our asymmetry measures capture higher-order asymmetry, which is associated with a lower expected return. In this case, although skewness offers no insights into future returns, our measures provide additional information beyond what can be inferred from skewness alone. Using the Fama–MacBeth approach, we find empirical evidence that higher skewness does not imply lower cross-sectional average returns for the universe of stocks up to Dec. 2015, which is consistent with Bali et al. (2011). In contrast, based on our new measures, we find that asymmetry

[1]Ghysels, Plazzi, and Valkanov (2016) provide an interesting quantile-based measure of conditional skewness, and the average cross-sectional correlations between our asymmetry proxies and their quantile-based skewness measure are around 0.50. However, high conditional skewness does not imply lower average returns for stocks, as shown later.

[2]Our article is about univariate asymmetry. Backus, Boyarchenko, and Chernov (2018), Chabi-Yo and Colacito (2019), and Bakshi and Chabi-Yo (2019) use entropy to measure general codependence between two variables; Jiang, Wu, and Zhou (2018) use entropy to measure bivariate asymmetry.

does help explain the cross-sectional variation in stock returns. The greater the upside asymmetry, the lower are the average returns in the cross-section. As an alternative approach, we sort stocks into deciles based on skewness as well as our asymmetry measures. We find that although high-skewness portfolios do not necessarily imply low returns, high upside asymmetries based on our measures are associated with low returns. Overall, we find that our measures explain the returns well and serve as useful complementary measures of asymmetry.

To examine the robustness of our results, we consider a number of controls: the tail-risk factor introduced by Kelly and Jiang (2014), the maximum and minimum returns suggested by Bali et al. (2011), and commonly used financial distress proxies. To guard against measurement error, we also smooth our estimated asymmetry measures via moving averages or by using portfolios rather than individual stocks as test assets. Our results and conclusions remain the same.

In an additional analysis, we investigate why the empirical evidence on the relation between expected returns and skewness is mixed. Following Boyer et al. (2010), we examine the association between these two variables while controlling for volatility. Interestingly, we find that skewness, the third moment, is closely related to volatility in exerting its impact on expected returns. When controlling for market volatility, skewness negatively affects returns only in high-volatility periods. When controlling for idiosyncratic volatility (IVOL), skewness negatively affects returns only for stocks with high IVOL. In contrast, neither market volatility nor IVOL significantly alters the effects of our asymmetry measures on stock returns.

We also examine the relationship between asymmetry and returns conditional on investor sentiment (Baker and Wurgler (2006)), market liquidity (Pástor and Stambaugh (2003)), and capital gains overhang (Grinblatt and Han (2005)), respectively. Following Stambaugh, Yu, and Yuan (2012), we run regressions of stock returns on skewness for high- and low-sentiment periods separately. We find that skewness has a significant negative association with stock returns in high-sentiment periods but a significant positive association with stock returns in low-sentiment periods. In contrast, using our measures of asymmetry, we find that upside asymmetry is negatively related to stock returns in both high- and low-sentiment periods. Further, when we control for either aggregate stock market liquidity or the capital gains overhang, we find similarly ambiguous results for skewness, which sheds light on why empirical evidence on the relation between expected returns and skewness is mixed. In comparison, we find consistent results for our measures when controlling for either aggregate stock market liquidity or the capital gains overhang.

The remainder of the article is organized as follows: Section II presents our new asymmetry measures. Section III applies these measures to test the symmetry of simulated data, size portfolios, and individual stocks and compares them to skewness. Section IV provides the major empirical results. Section V examines the asymmetry measures' relations with volatility, sentiment, market liquidity, and capital gains overhang. Section VI concludes.

II. Asymmetry Measures

In this section, we first introduce our asymmetry measures and discuss their properties. We then provide econometric procedures for their estimation in practice.

A. The First Measure

Without loss of generality, we assume that a stock return x is standardized with 0 mean and unit variance. Note that, following related empirical studies, we later focus on the idiosyncratic asymmetry of daily returns. In this case, x is the return residual after adjusting for the market risk. To assess the upside asymmetry of x, we consider its excess tail probability (ETP),

$$\text{(1)} \qquad \begin{aligned} \mathrm{IE}_{\varphi} &= \int_{1}^{+\infty} f(x)\,dx - \int_{-\infty}^{-1} f(x)\,dx \\ &= \int_{1}^{\infty} [f(x) - f(-x)]\,dx, \end{aligned}$$

where the probabilities are evaluated at 1 standard deviation away from the mean.[3] Its first term measures the cumulative chance of large gains, whereas its second term measures the cumulative chance of large losses. If IE_{φ} is positive, the probability of large gains is higher than the probability of large losses.

It is important to understand why IE_{φ} is related to a stock's expected return. Intuitively, everything else equal, investors prefer large gains or avoid large losses. As a result, they bid up the prices of stocks with large gains and pay low prices for stocks with large losses. In equilibrium, this implies lower expected returns when IE_{φ} is greater. Under albeit certain restrictive conditions, Appendix A provides an analytical proof of this point.

In short, consistent with theoretical studies such as Han et al. (2018), IE_{φ} captures upside asymmetry, and under certain conditions, a greater positive asymmetry is associated with a lower expected return. In practice, whether this claim is true is an empirical question. We confirm this association later in the article.

B. The Second Measure

Our second asymmetry measure is an entropy-scaled version of the first. Following Racine and Maasoumi (2007), consider a generic stationary series $\{X_t\}_{t=1}^{T}$ with mean $\mu_x = E[X_t]$ and density function $f(x)$. Let $\tilde{X}_t = -X_t + 2\mu_x$ be a rotation of X_t about its mean, and let $f(\tilde{x})$ be its density function. We say $\{X_t\}_{t=1}^{T}$ is symmetric around the mean if

$$\text{(2)} \qquad f(x) \equiv f(\tilde{x})$$

holds almost surely. Any difference between $f(x)$ and $f(\tilde{x})$ can be used to measure the magnitude of asymmetry. Following Racine and Maasoumi (2007),

[3] Our empirical results remain qualitatively similar when we shift our threshold slightly (e.g., from 1 standard deviation to 1.5 standard deviations). However, the sample size drastically decreases with a threshold of 2 or 3 standard deviations, yielding large estimation errors and insignificant results.

we define

$$(3) \qquad \mathrm{IS}_{\rho} = \frac{1}{2}\int_{-\infty}^{\infty}\left[f(x)^{1/2} - f(\tilde{x})^{1/2}\right]^2 dx.$$

For symmetric distributions, such as a normal distribution, IS_{ρ} is exactly 0. In general, the greater the asymmetry, the higher the value of IS_{ρ}.

We use a square-root transformation of the density function for IS_{ρ} following the definition proposed by Granger, Maasoumi, and Racine (2004). The measure is a special case of a general symmetric k-class entropy, taking $k=1/2$.[4] It can be shown that when $k=1/2$, this entropy is the only metric entropy within the symmetric k-class entropy family because it satisfies the triangular inequality.[5] Statistically, IS_{ρ} has four desirable properties. First, it can be applied to both discrete and continuous variables. Second, if $f(x)=f(\tilde{x})$ (i.e., the original and rotated distributions are equal), then $\mathrm{IS}_{\rho}=0$. Because of the normalization, the measure always lies between 0 and 1. Third, it is a metric, implying that a larger IS_{ρ} indicates a greater distance and that the measure is comparable across distributions. Finally, it is invariant under a continuous and strictly increasing transformation of the underlying variables.

Note that IS_{ρ} is a purely statistical measure of asymmetry, and it is constructed to be suitable for testing the asymmetry of a distribution. To link it to IE_{φ}, we need to use the sign of IE_{φ} to differentiate between upside and downside asymmetry because IS_{ρ} is always nonnegative. Upside or downside asymmetry makes no difference for IS_{ρ}, but these concepts are totally different economically. Hence, we define our second measure of asymmetry as follows:

$$(4) \qquad \mathrm{IS}_{\varphi} = \mathrm{sign}(\mathrm{IE}_{\varphi}) \times \int_{1}^{\infty}\left[f(x)^{1/2} - f(-x)^{1/2}\right]^2 dx.$$

Mathematically, IS_{φ} is closely related to IE_{φ}. In general, the greater the IS_{φ}, the greater the IE_{φ}. IE_{φ} provides an equal-weighted assessment of asymmetry, but IS_{φ} weights asymmetry by probability mass. Theoretically, IS_{φ} is a distance measure between the densities, whereas IE_{φ} is not. Empirically, the performances of IE_{φ} and IS_{φ} can vary, but they appear to be quite similar in all of our applications.

C. Estimation

In order to accurately estimate IE_{φ} and IS_{φ}, we next implement our analysis using up to 1 year of daily return observations.

The estimation of IE_{φ} is trivial because we can simply estimate the probabilities using the empirical distribution function. However, the estimation of IS_{φ} requires a substantial amount of computation. Following Maasoumi and Racine (2008), we use the Parzen–Rosenblatt kernel density estimator:

$$(5) \qquad \hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} k\left(\frac{X_i - x}{h}\right),$$

[4]When $k=1$, the entropy is well known as the Kullback–Leibler divergence measure (Kullback and Leibler (1951)).

[5]Please see Granger et al. (2004) for more details about k-class entropy.

where n is the sample size of the time-series data $\{X_i\}$, $k(\cdot)$ is a nonnegative and bounded kernel function such as the normal density, and h is a smoothing parameter or bandwidth, determined next.

In selecting the optimal bandwidth for equation (5), we use the well-known Kullback–Leibler likelihood cross-validation method (see Li and Racine (2007) for details). This procedure minimizes the Kullback–Leibler divergence between the actual density and its estimated counterpart:

$$(6) \qquad \max_h \mathscr{L} \quad = \quad \sum_{i=1}^{n} \ln\left[\hat{f}_{-i}(X_i)\right],$$

where $\hat{f}_{-i}(X_i)$ is the leave-one-out kernel estimator of $f(X_i)$, which is defined as follows:

$$(7) \qquad \hat{f}_{-i}(X_i) \quad = \quad \frac{1}{(n-1)h}\sum_{j\neq i}^{n} k\left(\frac{X_i - X_j}{h}\right).$$

It is well known that under the stationarity assumption, the estimated density converges to the true density. With the previous calculations, we can estimate IS_φ easily in practice by computing the associated integrals numerically.

III. Symmetry Tests

In this section, in order to gain further insights into the differences between skewness and our measures of asymmetry, we employ the skewness (ISKEW) and IS_ρ as test statistics of symmetry using simulated data, portfolios formed by firm size, and the returns of individual stocks from the U.S. stock market. We show that our measures can capture asymmetry undetected by skewness.

Many commonly used skewness tests, such as the test developed by D'Agostino (1970), assume normality in the null hypothesis. Therefore, they are mainly tests of normality and may reject the null when the data are symmetric but not normally distributed. Because we are interested in testing for return asymmetry rather than normality, it is inappropriate to apply these tests in our context. Instead, we need a test that makes no assumptions about the underlying distribution. A bootstrap-based skewness test achieves this goal. Under standard regularity conditions, inferences can be drawn based on a t-type statistic: $\hat{t}_{\text{SKEW}} = \widehat{\text{SKEW}}/\hat{\sigma}_{\text{SKEW}}$, where $\widehat{\text{SKEW}}$ is the sample skewness, and $\hat{\sigma}_{\text{SKEW}}$ is the standard error obtained via a nested resampling method as suggested by Racine (1997). As shown by Horowitz (2001), the bootstrap method with this t-type statistic achieves asymptotic refinements over the first-order asymptotic approximation. Monte Carlo simulations show that this test has very good finite-sample size and power properties.[6]

Following Racine and Maasoumi (2007), we construct the sampling distribution under the null via a stationary bootstrap resampling method. In particular, we use the same simulation setup as in the skewness test. Because IS_ρ is estimated using a nonparametric kernel density method, carrying out this test with a large

[6]The detailed simulation results are available from the authors.

number of bootstrap resamplings is computationally intensive. Following the suggestions of Racine and Maasoumi (2007), we determine the significance levels of the test via a stationary block bootstrap with 399 replications. It is adequate for our purposes because perturbations around 399 make nearly no difference to the results.

Consider first the case in which skewness is a good measure of asymmetry. We independently simulate samples of equal size ($n = 1,500$) from two distributions: $N(120, 240)$ and $\chi^2(10)$. The first is a normal distribution (symmetric) with a mean of 120 and a variance of 240, and the second is a χ^2 distribution (asymmetric) with 10 degrees of freedom. With 399 bootstrap resamplings, Panel A in Table 1 reports the test statistics and p-values for ISKEW and IS_ρ. With both measures, we do not reject the null for the normal distribution, but we do reject it for the χ^2 distribution at the 5% level of statistical significance. Hence, both measures work quite well in these two simple cases.

TABLE 1
Symmetry Tests

Panels A and B of Table 1 report symmetry test statistics and their associated p-values for simulated data and for both value- and equal-weighted size decile portfolios, respectively. Panel C provides firm characteristics for stocks rejected by symmetry tests. Variable definitions are provided in Appendix B. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Simulations

	$N(120, 240)$	$\chi^2(10)$	Beta(1,3.7) – Beta(1.3,2.3)
ISKEW	0.052	0.821***	0.030
p-value	(0.441)	(0.000)	(0.657)
$\text{IS}_\rho \times 100$	0.172	3.951***	0.351***
p-value	(0.128)	(0.000)	(0.003)

Panel B. Size Portfolios

	Value Weighted				Equal Weighted			
Portfolios	ISKEW	p-Value	$\text{IS}_\rho \times 100$	p-Value	ISKEW	p-Value	$\text{IS}_\rho \times 100$	p-Value
Size 1	0.944**	0.023	1.263***	0.000	1.307***	0.005	2.334***	0.000
Size 2	0.796	0.226	1.151***	0.000	1.080	0.185	1.088***	0.003
Size 3	0.426	0.243	0.769**	0.030	0.930	0.120	0.683*	0.063
Size 4	0.207	0.607	0.259	0.454	0.743	0.308	0.448	0.228
Size 5	0.019	0.932	0.389	0.343	1.095	0.248	0.390	0.506
Size 6	0.098	0.564	0.441*	0.085	0.688	0.143	0.589	0.125
Size 7	0.746	0.185	0.474	0.246	0.936**	0.038	0.651	0.140
Size 8	0.680	0.341	0.315	0.722	0.349	0.125	0.575	0.286
Size 9	0.370	0.303	0.419	0.266	1.154	0.378	0.190	0.927
Size 10	–0.186	0.717	0.274	0.333	–0.766	0.160	0.742	0.221

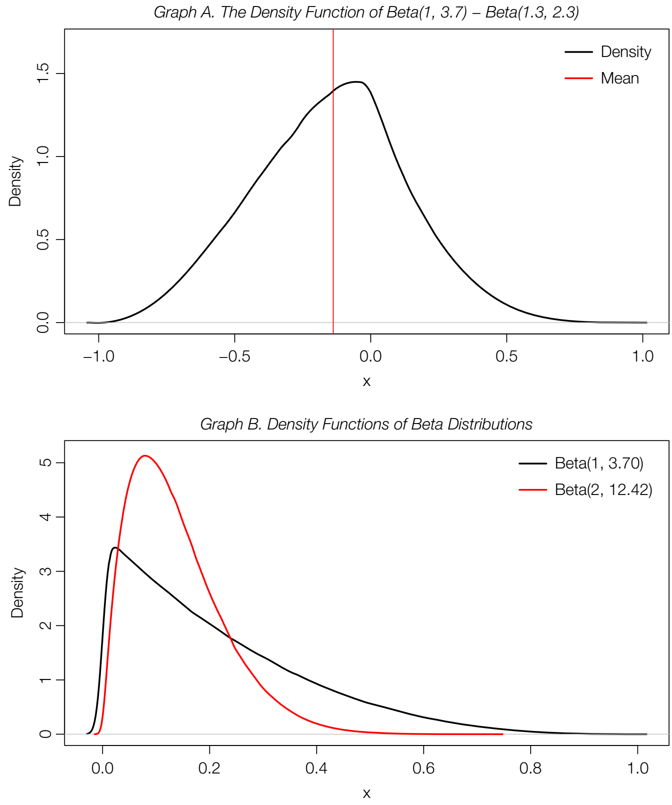
Panel C. Individual Stocks and Their Characteristics

	MAX	PRC	IVOL	ISKEW	TURN
Group 1 (no rejection by ISKEW test)	5.472	25.148	2.350	0.452	1.243
Group 2 (rejection by ISKEW test)	6.647	21.371	2.705	0.739	1.386
Group 3 (no rejection by IS_ρ test)	5.305	25.553	2.303	0.406	1.197
Group 4 (rejection by IS_ρ test)	6.808	21.639	2.732	0.812	1.522

Now consider a more complex situation: a beta distribution resulting from differencing two other beta distributions, Beta(1, 3.7) − Beta(1.3, 2.3). As illustrated in Graph A of Figure 1, this distribution has a longer left tail and shows

FIGURE 1
Examples of Asymmetric Distributions

Figure 1 shows plots of distribution-density functions. Graph A shows an asymmetric distribution with 0 skewness: the density function of Beta(1, 3.7) − Beta(1.3, 2.3). Graph B shows two Beta distributions with skewness equal to 1: Beta(1, 3.70) and Beta(2, 12.42).



negative asymmetry.[7] With the same sample size of 1,500 and 399 bootstrap resamples, the skewness test is now unable to detect any asymmetry. Indeed, column 4 of Panel A in Table 1 shows that this distribution has a skewness of 0.030 with a p-value of 0.657. In contrast, IS_ρ has a significant positive value of 0.351% and a p-value of 0.003, capturing the asymmetry of the distribution and rejecting the null hypothesis of symmetry at the 1% level.

In fact, it is not difficult to come up with two distributions with the same skewness but different levels of asymmetry. Graph B of Figure 1 plots two beta distributions: Beta(1, 3.70) and Beta(2, 12.42). They both have roughly the same skewness, but it is clear from Graph B that Beta(1, 3.70) has a longer right tail and a higher level of asymmetry. The skewness measure misses this difference, whereas IS_ρ captures it. Note that it is impossible to find a case where the opposite

[7]This is a well-defined distribution whose density function is provided by Pham-Gia, Turkkan, and Eng (1993) and Gupta and Nadarajah (2004).

is true: No pair of distributions has the same asymmetry but different skewness measures. This is the case because IS_ρ compares densities at every point in the upside and downside of the distribution and is thus more powerful than skewness in capturing asymmetry.

Second, we compare the performance of the distributional asymmetry measure IS_ρ with that of ISKEW using real data. We rely on the commonly used size decile portfolios sorted by market capitalization from Kenneth French's Web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) because, based on previous research, small stocks tend to be asymmetrically distributed (see Kumar (2009), and Bali et al. (2011)). We consider both value-weighted and equal-weighted portfolio returns. The sample period is from Jan. 1963 to Dec. 2015.

Panel B in Table 1 reports the results for both ISKEW and IS_ρ. For the value-weighted size portfolios, the IS_ρ test rejects symmetry for the three smallest portfolios at the conventional 5% level. In contrast, the skewness test can only detect asymmetry for the smallest portfolio at the 5% level. Similarly, for the equal-weighted size portfolios, the three smallest portfolios are asymmetric based on the IS_ρ test at 10%. In contrast, according to the skewness test, only the smallest and the 7th-smallest portfolios have significant asymmetry.

We next test for asymmetry using ISKEW and IS_ρ for individual stocks with at least 100 daily returns available per year during the sample period from Jan. 1963 to Dec. 2015. We exclude stocks with prices below $5 at the beginning of each month, leaving 159,356 stock-year paired observations in our sample. At the 5% significance level, the ISKEW test detects asymmetry for 17,838 observations (11.19%), whereas IS_ρ detects asymmetry for 29,348 observations (18.42%).

It is of interest to examine the characteristics of the asymmetric firms. To do so, we group all stock-year pairs into portfolios based on whether the null hypothesis is rejected by the ISKEW or IS_ρ tests at 5%. For brevity, we consider only 5 stock characteristics that are related to lottery preference (i.e., maximum return (MAX), price at the beginning of a month (PRC), idiosyncratic volatility (IVOL), idiosyncratic skewness (ISKEW), and turnover ratio (TURN)).[8] Panel C of Table 1 reports the empirical results. The average firm characteristics shown for group 1 (no rejection by the ISKEW test) versus group 2 (rejection by the ISKEW test) and for group 3 (no rejection by the IS_ρ test) versus group 4 (rejection by the IS_ρ test) are dramatically different from each other. Based on Kumar (2009) and Bali et al. (2011), we know that lottery-type stocks tend to have higher maximum daily returns, lower prices, higher idiosyncratic volatility, higher idiosyncratic skewness, and higher turnover ratios. Stocks identified by our entropy measure as asymmetric (group 4) match those characteristics better than all other groups.

[8]The definitions and computations of these variables and many later in the article, which are standard, are provided in Appendix B.

In summary, we find that our measures are valuable alternatives to skewness in measuring the asymmetry of stock returns. Although skewness can sufficiently describe asymmetry in some situations, it fails in others. In contrast, our measures consistently detect asymmetry more effectively, as evidenced by both simulations and real data.

IV. Empirical Results

A. Data

We use return data from the Center for Research in Security Prices (CRSP) from Aug. 1963 to Dec. 2015. The data include all common stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotations (NASDAQ). As before, we restrict the sample to stocks with beginning-of-month prices of $5 or more. To mitigate the concern of double-counted trading volume on NASDAQ, we follow Gao and Ritter (2010) and adjust the trading volume used to calculate the turnover ratio (TURN) and the Amihud (2002) ratio (ILLIQ). The latter is normalized to account for inflation and is truncated at 30 to eliminate the effect of outliers (Acharya and Pedersen (2005)). Firm size (SIZE), book-to-market ratio (BM), and momentum (MOM) are computed following Fama and French (1992) and Jegadeesh and Titman (1993). Market beta (β) is estimated by using the time-series regression of daily excess returns (in excess of the 1-month T-bill rate) on market excess returns. We use excess returns and risk-adjusted returns (adjusted for the Fama–French 3 factors) over the prior month as proxies for short-term reversals (REV for excess returns and REVA for risk-adjusted returns).

Following Bali et al. (2011), we compute the volatility (VOL), maximum (MAX), and minimum (MIN) of stock returns as the standard deviation, the maximum, and the negative of the minimum of daily returns throughout the month. In addition, we compute the idiosyncratic volatility (IVOL) of a stock as the standard deviation of daily idiosyncratic returns throughout the month. Following Ghysels et al. (2016), we calculate alternative estimators of idiosyncratic skewness: ISK_{INT} and ISK_{α}. We follow Harvey and Siddique (2000) and Chabi-Yo (2012) to estimate coskewness (COSKEW), and cokurtosis (COKURT) is obtained following Dittmar (2002) and Chabi-Yo (2012).

Because financial distress may be related to asymmetry, we control for its effects by using two proxies. O_SCORE represents the probability of bankruptcy; its calculation is based on a static model following Ohlson (1980). P_CHS is the probability of bankruptcy estimated from a dynamic logit model following Campbell, Hilscher, and Szilagyi (2008), (2011).

We use BW to denote the Baker and Wurgler (2006), (2007) sentiment index from Jeffrey Wurgler's Web site (http://people.stern.nyu.edu/jwurgler/). VIXM is the monthly variance of the daily value-weighted market return. Aggregate stock market liquidity (ALIQ) is provided by Pástor and Stambaugh (2003) (http://faculty.chicagobooth.edu/lubos.pastor/research/). Following Grinblatt and Han (2005), we calculate the capital gains overhang (CGO) for representative investors for each month using weekly prices and turnover ratios. The CGO at week t is the difference between the price at week $t-1$ and the reference price at week t (divided by the price at week $t-1$).

We examine the time-series averages of the cross-sectional correlations between estimated skewness, the alternative skewness estimator ISK_{INT}, volatility, and our asymmetry measures. Idiosyncratic skewness has small correlations with our measures, IE_{φ} and IS_{φ}. As expected, IE_{φ} and IS_{φ} have a high correlation of 68%, and they are also highly correlated with ISK_{INT}, which indicates that these alternative measures of skewness may have some similarities. The volatility has an approximately 8% correlation with the skewness and even lower correlations with IE_{φ} and IS_{φ}. The correlation analysis shows that our new asymmetry measures capture information beyond that included in volatility and skewness.

Panel A of Table 2 presents the summary statistics for the stocks in the deciles formed every month based on an ascending sort by IE_{φ} from Aug. 1963 to Dec. 2015. The panel reports the time-series averages of the median values of various asymmetry proxies for the stocks, namely, IE_{φ}, IS_{φ}, ISKEW, $\text{ISK}_{0.75}$, $\text{ISK}_{0.90}$, and ISK_{INT}. Both our asymmetry proxies and the alternative skewness measures provided by Ghysels et al. (2016) increase from the low- to the high-IE_{φ} portfolio, whereas ISKEW follows the opposite pattern. Furthermore, for stocks with the lowest upside asymmetry, ISKEW is positive but becomes negative when truncating the tails at the 75th percentile ($\text{ISK}_{0.75}$) or the 90th percentile ($\text{ISK}_{0.90}$).

B. Asymmetries and Firm Characteristics

We examine the types of stocks that are associated with asymmetry as measured by ISKEW, IE_{φ}, and IS_{φ} in Panel B of Table 2. Using idiosyncratic asymmetry measures as dependent variables, we run Fama–Macbeth regressions on the firm characteristics of SIZE, BM, MOM, TURN, ILLIQ, and the market beta (β):

$$\text{IA}_{i,t} \quad = \quad a_t + B_t X_{i,t} + \varepsilon_{i,t}, \tag{8}$$

where $\text{IA}_{i,t}$ is one of the 3 asymmetry measures for firm i at month t, and $X_{i,t}$ represents firm characteristics in the same month. Idiosyncratic asymmetry measures are winsorized at the 0.5th percentile and the 99.5th percentile. The Fama–MacBeth standard errors are adjusted using the Newey and West (1987) correction with 3 lags.[9] Consistent with other studies, such as Boyer et al. (2010) and Bali et al. (2011), ISKEW is negatively related to SIZE, BM, and TURN and positively related to MOM, ILLIQ, and market beta (β). IE_{φ} and IS_{φ} are significantly related to all firm characteristics; these relations all have the same direction as the relation between firm characteristics and skewness, except TURN. The correlation of our asymmetry measures with the turnover ratio matches Kumar (2009), who finds that lottery-type stocks have much higher turnover ratios than do other stocks.

C. Asymmetries and Expected Returns

In this subsection, we examine the ability of our new asymmetry measures to explain the cross section of stock returns and then compare them with skewness.

[9]All results are qualitatively similar if we use up to 24 lags.

TABLE 2

IE_{φ} Decile Portfolios, Firm Characteristics, and Asymmetry Measures

Panel A of Table 2 reports asymmetry measures for decile portfolios formed by sorting stocks based on IE_{φ} every month from Aug. 1963 to Dec. 2015. Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) IE_{φ}. Panel A reports the time-series average of the median values of various asymmetry proxies for the stocks: IE_{φ}, IS_{φ}, ISKEW, $ISK_{0.75}$, $ISK_{0.90}$, and ISK_{INT}. Panel B reports the average slopes and associated t-values (given in parentheses) of the Fama–MacBeth regressions

$$(9) \qquad IA_{i,t} = a_t + B_t X_{i,t} + \varepsilon_{i,t},$$

where $IA_{i,t}$ is ISKEW, IE_{φ}, or IS_{φ} at time t for stock i, and $X_{i,t}$ is a set of variables representing firm characteristics evaluated at the same time period t, including size (SIZE), book-to-market ratio (BM), momentum (MOM), turnover (TURN), illiquidity measure (ILLIQ), and market beta (β). The slopes are scaled by 100 for ease of reading. *** indicates significance at the 1% level.

Panel A. Different Asymmetry Proxies for Decile Portfolios Sorted by IE_{φ}

Decile	IE_{φ}	IS_{φ}	ISKEW	$ISK_{0.75}$	$ISK_{0.90}$	ISK_{INT}
1 (lowest)	−0.021	−0.040	0.587	−0.221	−0.149	−0.152
2	−0.012	−0.026	0.506	−0.055	0.016	0.012
3	−0.007	−0.021	0.459	0.049	0.107	0.106
4	−0.003	−0.018	0.418	0.129	0.178	0.178
5	0.001	0.009	0.379	0.208	0.240	0.241
6	0.004	0.019	0.349	0.280	0.302	0.304
7	0.008	0.021	0.317	0.357	0.368	0.372
8	0.012	0.024	0.282	0.439	0.438	0.444
9	0.018	0.031	0.242	0.533	0.535	0.541
10 (highest)	0.028	0.049	0.176	0.674	0.716	0.723

Panel B. Firm Characteristics and Asymmetry Measures

	ISKEW	IE_{φ}	IS_{φ}
SIZE	−9.111***	−0.025***	−0.113***
	(−47.33)	(−14.44)	(−19.36)
BM	−4.179***	−0.071***	−0.207***
	(−13.29)	(−25.17)	(−22.62)
MOM	0.808***	0.001***	0.008***
	(43.85)	(10.78)	(22.69)
TURN	−1.141***	0.131***	0.300***
	(−3.87)	(48.84)	(37.53)
ILLIQ	1.016***	0.011***	0.022***
	(6.52)	(5.64)	(2.42)
β	1.840***	0.050***	0.315***
	(2.86)	(9.72)	(16.37)
Constant	82.386***	0.152***	0.545***
	(61.36)	(12.11)	(15.10)
R^2	0.103	0.028	0.020

Like most antecedents, such as Ang, Hodrick, Xing, and Zhang (2006), Boyer et al. (2010), Bali et al. (2011), and Conrad et al. (2013), we focus on idiosyncratic asymmetry at the firm level. We run the following Fama–MacBeth regressions:

$$(10) \qquad R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t}\mathrm{IA}_{i,t} + \lambda_{2,t}\mathrm{ISKEW}_{i,t} + \Lambda_t X_{i,t} + \varepsilon_{i,t+1},$$

where $R_{i,t+1}$ is the excess return (the difference between the monthly stock return on stock i and the 1-month T-bill rate at time $t+1$); $\mathrm{IA}_{i,t}$ is either $\mathrm{IE}_{\varphi,i,t}$ or $\mathrm{IS}_{\varphi,i,t}$ for stock i at time t; and $X_{i,t}$ is a set of control variables including SIZE, BM, MOM, TURN, ILLIQ, β, MAX, REV, IVOL, COSKEW, and COKURT.

Columns 1–7 of Table 3 report the results. When regressing excess returns on either $\mathrm{IE}_{\varphi,i,t}$ or $\mathrm{IS}_{\varphi,i,t}$ alone, their coefficients are −3.866 and −0.863, respectively.

TABLE 3
Asymmetries and Expected Returns

Table 3 reports the time-series averages of the slope coefficients and their t-values (given in parentheses) from the Fama–MacBeth regressions of excess stock returns or risk-adjusted stock returns on various pricing variables (see column 1) using monthly data t ($t+1$) from July (Aug.) 1963 to Nov. (Dec.) 2015.

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t}\text{IA}_{i,t} + \lambda_{2,t}\text{ISKEW}_{i,t} + \Lambda_t X_{i,t} + \varepsilon_{i,t+1}, \quad (11)$$

where $R_{i,t+1}$ is the excess return, which is the difference between the monthly stock return on stock i and the 1-month T-bill rate at time $t+1$ or the risk-adjusted return on stock i at $t+1$, which is adjusted for the Fama–French 3 factors; $\text{IA}_{i,t}$ is either $\text{IS}_{\varphi,i,t}$ or $\text{IE}_{\varphi,i,t}$ for stock i at time t; and $X_{i,t}$ is a set of control variables. For columns 1–7, the dependent variable is the excess return (R). The risk-adjusted return (RA) is the dependent variable for columns 8–14. We adjust the Fama–MacBeth standard errors using the Newey and West (1987) correction with three lags. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	R	*R*	*R*	*R*	*R*	*R*	*R*	RA	RA	RA	RA	RA	RA	RA
ISKEW	0.012 (0.43)			0.004 (0.12)	−0.005 (−0.29)	0.012 (0.42)	0.002 (0.10)	−0.020 (−1.08)			−0.027 (−1.38)	−0.015 (−0.94)	−0.022 (−1.12)	−0.011 (−0.68)
IE_φ		−3.866*** (−2.91)		−4.102*** (−2.88)	−3.286*** (−4.18)				−2.862*** (−3.24)		−3.232*** (−3.49)	−2.567*** (−3.60)		
IS_φ			−0.863*** (−2.63)			−0.894*** (−2.74)	−0.738*** (−3.51)			−0.695*** (−3.21)			−0.759*** (−3.39)	−0.667*** (−3.41)
SIZE					−0.280*** (−8.21)		−0.281*** (−8.23)					−0.245*** (−12.51)		−0.246*** (−12.54)
BM					0.185*** (3.52)		0.185*** (3.52)					0.017 (0.47)		0.017 (0.48)
MOM					0.008*** (5.62)		0.008*** (5.59)					0.008*** (5.65)		0.008*** (5.63)
TURN					0.004 (0.13)		0.003 (0.11)					0.001 (0.03)		0.001 (0.05)
ILLIQ					0.035** (2.40)		0.035** (2.41)					0.043*** (2.87)		0.043*** (2.88)
β					0.285 (1.09)		0.291 (1.11)							
MAX					0.037*** (5.00)		0.035*** (4.73)					0.022*** (2.97)		0.020*** (2.69)
IVOL					−0.445*** (−14.05)		−0.444*** (−14.06)					−0.342*** (−12.49)		−0.341*** (−12.46)
COSKEW					−1.313*** (−3.87)		−1.315*** (−3.88)					−1.113*** (−3.61)		−1.114*** (−3.62)
COKURT					0.600*** (5.32)		0.598*** (5.30)					0.681*** (8.25)		0.682*** (8.26)
REV					−0.038*** (−10.09)		−0.038*** (−10.01)							
REVA												−0.044*** (−12.28)		−0.044*** (−12.21)
Constant	0.644*** (2.84)	0.664*** (2.88)	0.659*** (2.86)	0.657*** (2.91)	1.920*** (6.90)	0.648*** (2.87)	1.925*** (6.92)	0.056 (1.59)	0.056* (1.73)	0.054* (1.66)	0.066* (1.84)	1.059*** (8.53)	0.061* (1.73)	1.067*** (8.59)
R^2	0.003	0.002	0.001	0.005	0.102	0.004	0.102	0.002	0.001	0.001	0.003	0.049	0.003	0.049

Both are significant at the 1% level, and their signs are consistent with the theoretical prediction that upside asymmetry is negatively related to expected returns. These results are also economically significant: A 1-standard-deviation increase in asymmetry ($\text{IE}_{\varphi,i,t}$) drives down monthly average returns by approximately 5.74 basis points (bps). In contrast, the coefficient on ISKEW is slightly positive at 0.012 and is statistically insignificant. Hence, it is inconclusive whether realized skewness can explain the cross section of stock returns over the period from Aug. 1963 to Dec. 2015.[10]

The explanatory power of $\text{IE}_{\varphi,i,t}$ and $\text{IS}_{\varphi,i,t}$ is robust to various controls. For instance, adding ISKEW into the regression that includes $\text{IE}_{\varphi,i,t}$ changes the slope only slightly, from −3.866 to −4.102 (this new coefficient is also statistically

[10]Additionally, in Table IA.1 of the Supplementary Material, we show the results of applying alternative measures of skewness suggested by Ghysels et al. (2016). We find that cross-sectionally, the effect of these measures on the return is also mixed.

significant at the 1% level). Additional controls change neither the sign nor the significance level of the coefficient for $IE_{\varphi,i,t}$. Similar patterns hold true for $IS_{\varphi,i,t}$.

Because the Fama–French market, size (SMB), and book-to-market (HML) factors are often used to capture systematic risks, we test the robustness of our results by running our regressions with risk-adjusted returns as the dependent variable. We adjust returns by removing these systematic components and denote the risk-adjusted return of stock i as RA_i. We then rerun the earlier regressions from equation (10).

Columns 8–14 of Table 3 report the results. In this alternative model specification, skewness remains insignificant. In contrast, the effects of both $IE_{\varphi,i,t}$ and $IS_{\varphi,i,t}$ are significantly negative, as before. The results reaffirm that our new asymmetry measures have significant power in explaining the cross section of stock returns, whereas the skewness measure barely matters.[11]

Because our asymmetry measures are estimated rather than observed, there might be some concern regarding the estimation error. To alleviate this concern, we use two approaches to check the robustness of our conclusions in the Supplementary Material. The first approach is to smooth the estimates using moving averages,[12] and the second is to perform the previous Fama–MacBeth analysis at the portfolio level.[13]

To conduct the portfolio-level analysis, we first sort stocks into $5 \times 5 \times 5$ portfolios by size, book-to-market ratio, and momentum for each month. Then, we calculate portfolio asymmetry as equal-weighted averages of the asymmetry measures for all the stocks in the portfolio. Portfolio analyses tend to reduce the estimation error. When the number of stocks in the portfolio is large enough, the estimation error of the portfolio asymmetry measure should approach 0. Armed with these portfolio asymmetry measures, it is a straightforward process to rerun the previous Fama–MacBeth regressions using either excess portfolio returns or risk-adjusted returns. Table IA.6 of the Supplementary Material reports the results, which confirm that when considering portfolio asymmetry, our new asymmetry measures are again negatively related to expected returns. In contrast, skewness cannot explain the cross section of portfolio returns over the period Aug. 1963–Dec. 2015.

In an unreported table, we examine persistence through Fama–MacBeth regressions of asymmetry proxies on lagged values of asymmetry measured in the

[11]This conclusion holds with the expected skewness in Boyer et al. (2010) or Bali et al. (2011). See Table IA.2 of the Supplementary Material.

[12]For each stock, we first estimate asymmetry using daily returns over 3 months. We then take the moving averages of the past 4 quarters to calculate IE_{φ}^{MA} and IS_{φ}^{MA}. This tends to smooth out the estimation errors in each quarter. Table IA.3 of the Supplementary Material reports the results based on the moving-average estimates. Once again, we find that IE_{φ}^{MA} and IS_{φ}^{MA} are economically and statistically significant and negatively related to future returns, which is consistent with our previous findings.

[13]For additional robustness, we examine two other dimensions: i) adjusting the Fama–MacBeth standard errors using the Newey and West (1987) correction with 24 lags and ii) using daily return information over 6 rather than 12 months to estimate ISKEW, IE_{φ}, and IS_{φ}. We find qualitatively similar results (see Tables IA.4 and IA.5 of the Supplementary Material).

previous year (nonoverlapping period for the estimation) and other control variables. The coefficients of lagged IE_φ and IS_φ are positive and statistically significant at the 1% level, indicating that IE_φ and IS_φ are cross-sectionally persistent.

D. Asymmetry Portfolios

In Table 4, we examine the performance of 10 portfolios sorted by skewness, $IE_{\varphi,i,t}$, and $IS_{\varphi,i,t}$, respectively. We report the equal-weighted averages of stock returns for each decile and calculate the return spread between the highest- and lowest-asymmetry portfolios (decile 10 and decile 1, respectively). For skewness, the monthly excess returns display no monotonic pattern. The return difference between portfolios 1 and 10 is 0.078% per month, which is neither economically nor statistically significant. Hence, high skewness does not necessarily imply a low return. Columns 3 and 4 of Table 4 report the results based on the capital asset pricing model (CAPM) and Fama and French (1993) 3-factor alphas. The spread portfolio has a CAPM alpha of 0.084% per month and a Fama–French alpha of 0.061% per month, both of which are small and insignificant. The results show that a trading strategy based on skewness does not generate a significant profit.[14]

TABLE 4
Decile Portfolios

Table 4 reports the equal-weighted averages of monthly stock returns, the capital asset pricing model (CAPM) alpha, and the Fama–French 3-factor (FF3) alpha, as well as their *t*-values (given in parentheses), for decile portfolios sorted by ISKEW, IE_φ, and IS_φ in the previous month based on data t ($t+1$) from July (Aug.) 1963 to Nov. (Dec.) 2015. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	ISKEW			IE_φ			IS_φ		
Portfolio	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)
1 (lowest)	0.460**	−0.072	−0.238***	0.688***	0.198**	−0.015	0.745***	0.205**	0.008
	(2.19)	(−0.77)	(−3.63)	(3.51)	(2.16)	(−0.28)	(3.45)	(2.03)	(0.16)
2	0.631***	0.126	−0.054	0.694***	0.174*	−0.031	0.750***	0.223**	0.012
	(3.23)	(1.61)	(−1.02)	(3.39)	(1.95)	(−0.66)	(3.61)	(2.44)	(0.24)
3	0.637***	0.132*	−0.056	0.717***	0.191**	−0.003	0.690***	0.177**	−0.027
	(3.26)	(1.66)	(−1.10)	(3.49)	(2.20)	(−0.06)	(3.43)	(2.02)	(−0.55)
4	0.676***	0.160**	−0.027	0.717***	0.183**	−0.010	0.687***	0.187**	−0.013
	(3.38)	(1.97)	(−0.56)	(3.44)	(2.09)	(−0.22)	(3.51)	(2.24)	(−0.26)
5	0.741***	0.210**	0.034	0.694***	0.151*	−0.037	0.614***	0.098	−0.078*
	(3.60)	(2.48)	(0.71)	(3.29)	(1.73)	(−0.84)	(3.06)	(1.17)	(−1.72)
6	0.782***	0.234**	0.039	0.682***	0.131	−0.051	0.602***	0.061	−0.113***
	(3.64)	(2.51)	(0.83)	(3.19)	(1.49)	(−1.19)	(2.88)	(0.72)	(−2.59)
7	0.712***	0.148	−0.033	0.589***	0.037	−0.135***	0.617***	0.072	−0.097**
	(3.19)	(1.48)	(−0.68)	(2.75)	(0.42)	(−3.16)	(2.91)	(0.81)	(−2.18)
8	0.728***	0.145	−0.037	0.636***	0.081	−0.085**	0.617***	0.054	−0.116***
	(3.13)	(1.35)	(−0.73)	(2.93)	(0.88)	(−2.04)	(2.82)	(0.59)	(−2.60)
9	0.645***	0.061	−0.113**	0.588***	0.025	−0.133***	0.630***	0.055	−0.110**
	(2.73)	(0.53)	(−2.17)	(2.66)	(0.26)	(−3.06)	(2.79)	(0.56)	(−2.47)
10 (highest)	0.538**	0.013	−0.177***	0.503**	−0.062	−0.215***	0.557**	−0.024	−0.182***
	(2.45)	(0.11)	(−3.13)	(2.23)	(−0.60)	(−4.49)	(2.38)	(−0.21)	(−3.50)
10 − spread	0.078	0.084	0.061	−0.186***	−0.260***	−0.200***	−0.188***	−0.229***	−0.190***
	(0.82)	(0.89)	(0.70)	(−2.67)	(−4.02)	(−3.47)	(−3.35)	(−4.18)	(−3.69)

[14]This result is not confined to small stocks because sorting stocks first by firm size and then by asymmetry yields similar results (see Table IA.7 of the Supplementary Material).

Consider now the asymmetry measure $\mathrm{IS}_{\varphi,i,t}$. Column 8 of Table 4 clearly shows a decreasing pattern of returns across the decile portfolios. Moreover, the spread portfolio has a (negatively) large value of −0.188% per month, which is statistically significant at the 1% level. The annualized return of this portfolio is 2.26%, which is economically significant. In addition, the return difference cannot be explained by the Fama and French (1993) risk factors. Overall, there is strong evidence that a high $\mathrm{IS}_{\varphi,i,t}$ leads to a low return. Theoretically, the results are consistent with Tversky and Kahneman (1992), Han et al. (2018), and the illustration model presented in Appendix A, whose conclusions generally imply that greater upside asymmetry is associated with a lower expected return. Because high skewness does not always lead to high upside asymmetry, its empirical impact on the return remains unclear.

For measure $\mathrm{IE}_{\varphi,i,t}$, the decreasing pattern of returns across deciles is similar to that of $\mathrm{IS}_{\varphi,i,t}$, and the spread portfolio earns significant alphas.[15] This result is not surprising because the two measures are similar, and their average cross-sectional correlation with each other is approximately 68%. These results confirm the findings from the previous Fama–MacBeth regressions regarding the superior ability of the new asymmetry measures to explain the cross-section of stock returns.

Finally, it is of interest to examine what ranges of data are decisive for the results. For this purpose, we consider

(12)
$$\begin{aligned} \mathrm{IE}_{\varphi}^{b} &= \int_{1}^{b} f(x)\,dx - \int_{-b}^{-1} f(x)\,dx, \\ \mathrm{IS}_{\varphi}^{b} &= \operatorname{sign}\left(\mathrm{IE}_{\varphi}^{b}\right) \times \int_{1}^{b} \left[f(x)^{1/2} - f(-x)^{1/2} \right]^{2} dx. \end{aligned}$$

Panel A of Table 5 reports the portfolio results for $b=1.5$. It is interesting that the results are similar, although weaker, compared with Table 4. The spread portfolio based on $\mathrm{IS}_{\varphi}^{1.5}$ is still significant relative to $\mathrm{IE}_{\varphi}^{1.5}$, suggesting the value of using entropy in finance.

Moreover, with $b=2$, as reported in Panel B of Table 5, the evidence becomes stronger and is almost as good as that in Table 4, suggesting that data in the range from 1 standard deviation below/above the mean to 2 standard deviations below/above the mean are perhaps all one needs to determine the asymmetry effect.[16] In comparison, if one uses a skewness measure, the results are not significant at all. For example, for the data range from 1 standard deviation below/above the mean to 2 standard deviations below/above the mean, the spread portfolio based on skewness has an insignificant mean of −0.009 with a t-statistic of −0.11, suggesting the value of using our proposed asymmetry measures.

[15] The results are similar when applying Fama and French (2015) 5-factor models.

[16] We also find significant results for both measures for data in the range from 1.5 to 2 standard deviations below/above the mean (see Tables IA.8 and IA.9 of the Supplementary Material).

TABLE 5
Decile Portfolios

Table 5 reports the equal-weighted averages of monthly stock returns, the capital asset pricing model (CAPM) alpha, and the Fama–French 3-factor (FF3) alpha, as well as their t-values (given in parentheses), for decile portfolios sorted by $IE_{\varphi}^{1.5}$, $IS_{\varphi}^{1.5}$, IE_{φ}^{2}, and IS_{φ}^{2} in the previous month based on data t ($t+1$) from July (Aug.) 1963 to Nov. (Dec.) 2015 (using the observations from 1 to 1.5 standard deviations above and below the mean for Panel A and the observations from 1 to 2 standard deviations above and below the mean for Panel B to estimate $IE_{\varphi,i,t}$ and $IS_{\varphi,i,t}$).*, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Range from 1 to 1.5 Standard Deviations

	$IE_{\varphi}^{1.5}$			$IS_{\varphi}^{1.5}$		
Portfolio	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)
1 (lowest)	0.690***	0.127	−0.053	0.657***	0.101	−0.086*
	(3.01)	(1.12)	(−0.94)	(2.95)	(0.96)	(−1.76)
2	0.731***	0.161	−0.010	0.722***	0.164	−0.022
	(3.24)	(1.58)	(−0.21)	(3.26)	(1.62)	(−0.48)
3	0.674***	0.116	−0.069	0.713***	0.153	−0.022
	(3.06)	(1.18)	(−1.51)	(3.25)	(1.63)	(−0.51)
4	0.671***	0.118	−0.079*	0.623***	0.080	−0.117***
	(3.08)	(1.22)	(−1.73)	(2.92)	(0.86)	(−2.63)
5	0.678***	0.132	−0.051	0.691***	0.139	−0.041
	(3.18)	(1.46)	(−1.20)	(3.23)	(1.58)	(−0.93)
6	0.615***	0.076	−0.114**	0.671***	0.121	−0.061
	(2.94)	(0.89)	(−2.56)	(3.13)	(1.34)	(−1.41)
7	0.641***	0.106	−0.075*	0.606***	0.068	−0.106**
	(3.10)	(1.27)	(−1.75)	(2.90)	(0.79)	(−2.53)
8	0.641***	0.114	−0.061	0.662***	0.130	−0.053
	(3.15)	(1.40)	(−1.41)	(3.20)	(1.52)	(−1.15)
9	0.598***	0.082	−0.097**	0.617***	0.102	−0.075
	(2.99)	(1.00)	(−2.10)	(3.10)	(1.27)	(−1.59)
10 (highest)	0.569***	0.77	−0.106**	0.542***	0.037	−0.143***
	(2.95)	(0.93)	(−2.08)	(2.74)	(0.43)	(−2.93)
10 − 1 spread	−0.122	−0.050	−0.054	−0.115*	−0.065	−0.057
	(−1.54)	(−0.66)	(−0.83)	(−1.83)	(−1.07)	(−1.04)

Panel B. Range from 1 to 2 Standard Deviations

	IE_{φ}^{2}			IS_{φ}^{2}		
Portfolio	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)	Excess Return (%)	CAPM Alpha (%)	FF3 Alpha (%)
1 (lowest)	0.722***	0.177*	−0.013	0.679***	0.141	−0.056
	(3.29)	(1.67)	(−0.24)	(3.13)	(1.36)	(−1.12)
2	0.688***	0.132	−0.055	0.700***	0.151	−0.044
	(3.12)	(1.32)	(−1.16)	(3.22)	(1.55)	(−0.94)
3	0.734***	0.181*	−0.012	0.665***	0.118	−0.067
	(3.37)	(1.90)	(−0.27)	(3.11)	(1.29)	(−1.52)
4	0.642***	0.095	−0.087**	0.660***	0.112	−0.070
	(3.00)	(1.03)	(−1.99)	(3.08)	(1.24)	(−1.62)
5	0.661***	0.115	−0.075*	0.725***	0.168*	−0.016
	(3.10)	(1.28)	(−1.71)	(3.34)	(1.85)	(−0.36)
6	0.640***	0.099	−0.085**	0.657***	0.111	−0.066
	(3.04)	(1.13)	(−1.98)	(3.09)	(1.25)	(−1.57)
7	0.651***	0.115	−0.061	0.649***	0.111	−0.068
	(3.13)	(1.34)	(−1.38)	(3.10)	(1.27)	(−1.63)
8	0.659***	0.127	−0.056	0.639***	0.117	−0.068
	(3.18)	(1.46)	(−1.27)	(3.14)	(1.38)	(−1.49)
9	0.575***	0.042	−0.129***	0.584***	0.053	−0.123***
	(2.79)	(0.51)	(−2.96)	(2.84)	(0.64)	(−2.70)
10 (highest)	0.535***	0.027	−0.141***	0.547***	0.022	−0.144***
	(2.70)	(0.32)	(−2.91)	(2.67)	(0.25)	(−3.08)
10 – 1 spread	−0.187***	−0.150**	−0.128**	−0.131**	−0.119**	−0.088*
	(−2.96)	(−2.41)	(−2.21)	(−2.45)	(−2.21)	(−1.69)

V. Further Comparison of Asymmetry Measures

A. Asymmetry and Tail Risk, Extreme Returns, and Financial Distress

In this subsection, we examine whether our conclusions are sensitive to related variables such as tail risk, extreme returns, and firms' financial distress. First, we change the risk-adjusted return by controlling for Kelly and Jiang (2014)'s tail-risk factor. Second, in cross-sectional regressions, we control for MAX and MIN as in Bali et al. (2011). Lastly, we control for the financial distress measures O_SCORE (Ohlson (1980)) and P_CHS (Campbell et al. (2008), (2011)).

It is of interest to see how our asymmetry measures are related to tail risk, as defined by Kelly and Jiang (2014). We run the Fama–MacBeth regressions using equation (10) and the adjusted returns, which removes the tail risk in addition to the market, size (SMB), and book-to-market (HML) factors. Columns 1–5 of Table 6 show that the effects of both $IE_{\varphi,i,t}$ and $IS_{\varphi,i,t}$ on risk-adjusted returns remain significantly negative even when controlling for the tail-risk factor. The results suggest that upside asymmetry is priced in the cross section of stock returns and cannot be captured by the tail-risk factor.

TABLE 6
Fama–MacBeth Regressions

Table 6 reports the time-series averages of the slope coefficients and their t-values (given in parentheses) from Fama–MacBeth regressions on various pricing variables (see column 1) using monthly data t ($t+1$) from July (Aug.) 1963 to Nov. (Dec.) 2015.

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t}\text{IA}_{i,t} + \lambda_{2,t}\text{ISKEW}_{i,t} + \Lambda_t X_{i,t} + \varepsilon_{i,t+1}, \quad (13)$$

where $R_{i,t+1}$ is the excess return or risk-adjusted return of stock i at time $t+1$, $IA_{i,t}$ is either $IE_{\varphi,i,t}$ or $IS_{\varphi,i,t}$ at time t for stock i, and $X_{i,t}$ is a set of control variables. We adjust the Fama–MacBeth standard errors using the Newey and West (1987) correction with 3 lags. For columns 1–5, the dependent variable is the return adjusted for the Fama–French 3 factors and tail risk (RA). The excess return (R) is the dependent variable for columns 6–13. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	1	2	3	4	5	6	7	8	9	10	11	12	13
	RA	RA	RA	RA	RA	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
ISKEW	−0.031*			−0.022	−0.017			−0.019	−0.010			−0.034**	−0.026
	(−1.73)			(−1.33)	(−1.03)			(−1.09)	(−0.61)			(−1.97)	(−1.54)
IE_{φ}		−3.093***		−2.715***		−2.743**		−4.003***		−2.864***		−3.959***	
		(−3.60)		(−3.70)		(−2.51)		(−5.09)		(−2.82)		(−5.04)	
IS_{φ}			−0.749***		−0.659***		−0.585**		−0.884***		−0.714**		−0.908***
			(−3.63)		(−3.30)		(−2.19)		(−4.15)		(−2.58)		(−4.28)
SIZE				−0.128***	−0.129***			−0.196***	−0.196***			−0.190***	−0.191***
				(−9.39)	(−9.41)			(−4.98)	(−4.99)			(−4.82)	(−4.84)
BM				0.006	0.007			0.210***	0.211***			0.210***	0.210***
				(0.16)	(0.17)			(3.88)	(3.88)			(3.87)	(3.87)
MOM				0.008***	0.008***			0.009***	0.009***			0.009***	0.009***
				(6.30)	(6.28)			(5.96)	(5.93)			(6.04)	(6.01)
TURN				0.129***	0.129***			0.001	−0.000			0.014	0.013
				(3.83)	(3.83)			(0.03)	(−0.01)			(0.41)	(0.37)
ILLIQ				0.039***	0.039***			0.033**	0.034**			0.029**	0.029**
				(2.74)	(2.75)			(2.34)	(2.36)			(2.04)	(2.08)
MAX				0.032***	0.030***	−0.083***	−0.083***	0.037***	0.035***				
				(4.18)	(3.90)	(−6.91)	(−6.88)	(4.95)	(4.65)				
MIN										−0.060***	−0.061***	−0.091***	−0.089***
										(−3.28)	(−3.30)	(−10.02)	(−9.79)
β								0.745***	0.750***			0.766***	0.769***
								(3.71)	(3.73)			(3.79)	(3.81)
IVOL				−0.403***	−0.401***			−0.505***	−0.504***			−0.245***	−0.254***
				(−14.06)	(−14.02)			(−16.61)	(−16.66)			(−8.88)	(−9.25)
REVA4				−0.044***	−0.044***								
				(−12.15)	(−12.08)								
REV								−0.035***	−0.035***			−0.041***	−0.041***
								(−9.39)	(−9.32)			(−10.30)	(−10.24)
Constant	0.072**	0.063*	0.060*	1.248***	1.256***	1.120***	1.120***	2.028***	2.032***	0.949***	0.949***	2.004***	2.010***
	(2.10)	(1.96)	(1.88)	(11.07)	(11.13)	(5.88)	(5.88)	(7.16)	(7.18)	(5.37)	(5.38)	(7.04)	(7.07)
R^2	0.002	0.001	0.001	0.035	0.035	0.014	0.014	0.090	0.090	0.015	0.015	0.091	0.091

Our second question is whether asymmetry measures still predict returns after controlling for maximum (MAX) and minimum (MIN) returns over the previous month. Columns 6–13 of Table 6 report the Fama–MacBeth regression results. For all of the econometric specifications, the average coefficients on IE_{φ} and IS_{φ} remain negative and significant, suggesting that the effects of our asymmetry measures cannot be subsumed by extreme positive or negative returns.

Finally, we examine the effects of the asymmetry measures after controlling for O_SCORE and failure probability (P_CHS). Table IA.10 of the Supplementary Material reports the Fama–MacBeth regression results. Again, the average coefficients on IE_{φ} and IS_{φ} are always negative and significant at the 5% level, indicating that the substantial cross-sectional explanatory power of asymmetry is robust to controls for financial distress.

B. Asymmetry and Volatility

In this subsection, we examine how volatility impacts skewness and our new asymmetry measures by controlling for volatility in two ways. First, we define volatility regimes based on realized market volatility (VIXM).[17] Second, we use idiosyncratic volatility (IVOL) to define high- and low-IVOL stocks and control for IVOL to observe the effect of asymmetry measures on expected return.

High-volatility periods are defined as months in which the realized VIXM is above its mean, whereas low-volatility periods are defined as months in which the realized VIXM is below its mean. We use the same regressions of the excess returns on ISKEW and various controls as before:

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t}\text{ISKEW}_{i,t} + \Lambda_t X_{i,t} + \varepsilon_{i,t+1}, \quad (14)$$

where $X_{i,t}$ is the vector of control variables. Panel A of Table IA.11 reports the results. Skewness always has a significant negative effect on the expected return when VIXM is high, whether or not there are other controls in place. However, when VIXM is low, the coefficients on skewness are always positive. The contrasting signs of the coefficients on skewness during high- and low-volatility periods confirm that there is no consistent relationship between skewness and the cross-sectional average returns.

In comparison, when we repeat Fama–MacBeth regressions of excess returns on IE_{φ} and IS_{φ} conditional on market volatility, both always have negative slopes in both of the VIXM regimes, although the magnitudes and statistical significance vary (see Panels B and C of Table IA.11). In short, whereas skewness explains asset returns differently under various VIXM regimes, IE_{φ} and IS_{φ} provide consistent results regardless of whether VIXM is high or low.

We then conduct a sequential double-sorting analysis. At the beginning of each month from Aug. 1963 to Dec. 2015, we first sort stocks by IVOL into quintile portfolios. Then, within each IVOL portfolio, we further sort stocks into quintile portfolios by one of the following asymmetry measures: ISKEW, IE_{φ}, or IS_{φ}. Table 7 reports the equal-weighted averages of the excess returns of these portfolios. The negative average return generated by the spread portfolio (P5 − P1,

[17] Our findings are similar when applying the previous month's Chicago Board Options Exchange (CBOE) Volatility Index (VIX); the results are available from the authors.

TABLE 7
Portfolio Sorted by IVOL and Asymmetry Measures

Table 7 reports the average returns and associated *t*-values for quintile portfolios sorted by IVOL and then by ISKEW, IE_φ, or IS_φ from Aug. 1963 to Dec. 2015. IVOL1 and IVOL5 denote the lowest and highest quintiles for IVOL. P1 and P5 denote the lowest and highest quintiles for ISKEW, IE_φ, and IS_φ, respectively. *t*-statistics are given in parentheses. ** and *** indicate significance at the 5% and 1% levels, respectively.

	ISKEW			IE_φ			IS_φ		
Proxy	P1	P5	P5 – P1	P1	P5	P5 – P1	P1	P5	P5 – P1
IVOL1	0.581***	0.852***	0.271***	0.726***	0.773***	0.047	0.765***	0.792***	0.027
	(3.70)	(5.48)	(4.40)	(4.80)	(4.94)	(1.08)	(5.04)	(5.09)	(0.65)
IVOL2	0.814***	1.122***	0.308***	0.966***	0.900***	−0.067	1.023***	0.921***	−0.102**
	(4.28)	(5.63)	(4.27)	(5.17)	(4.72)	(−1.27)	(5.33)	(4.74)	(−2.07)
IVOL3	0.752***	0.957***	0.205**	0.968***	0.837***	−0.131**	1.026***	0.971***	−0.056
	(3.43)	(4.12)	(2.42)	(4.52)	(3.73)	(−2.00)	(4.63)	(4.21)	(−0.88)
IVOL4	0.639**	0.769***	0.130	0.797***	0.682***	−0.115	0.911***	0.671**	−0.240***
	(2.54)	(2.93)	(1.41)	(3.29)	(2.63)	(−1.62)	(3.58)	(2.56)	(−3.52)
IVOL5	0.025	−0.140	−0.164	0.087	−0.127	−0.214**	0.120	−0.053	−0.173**
	(0.09)	(−0.50)	(−1.42)	(0.32)	(−0.43)	(−2.42)	(0.43)	(−0.18)	(−2.10)
Avg(V1 – V5)	0.562***	0.712***	0.150**	0.709***	0.613***	−0.096**	0.769***	0.660***	−0.109***
	(2.64)	(3.28)	(2.46)	(3.45)	(2.83)	(−2.49)	(3.63)	(3.00)	(−3.27)

the return difference between the highest- and lowest-skewness stocks) only appears when stocks have high IVOL. Among the other 4 IVOL quintile portfolios, 3 ISKEW spread portfolios have significant positive returns, thus confirming that skewness is sensitive to the IVOL level. In contrast, the spread portfolios for IE_φ and IS_φ have mostly significant and negative returns across the IVOL quintiles. In summary, whereas the effect of skewness on the expected return depends on the second moment (in the form of both market volatility and stocks' IVOL), IE_φ and IS_φ are less sensitive to volatility.[18]

C. Sentiment

We next examine how skewness and asymmetry measures perform under varying levels of investor sentiment and aggregate stock market liquidity. We then study the measures' interaction with the capital gains overhang.

In this subsection, we examine how asymmetry effects vary during high- and low-sentiment periods. Stambaugh et al. (2012), Stambaugh, Yu, and Yuan (2015) find that anomalous returns are high during high-sentiment periods because mispricing is likely to be more prevalent when investor sentiment is high. It is of interest to investigate whether asymmetry effects on the expected return are related to sentiment.

Following Stambaugh et al. (2012), (2015), we run two separate Fama–MacBeth regressions. The first covers high-sentiment periods, which are defined here as months in which the Baker and Wurgler (2006) sentiment index (henceforth, BW index) is at least 1 standard deviation above its mean. The second includes low-sentiment periods, during which the BW index is at least 1 standard deviation below its mean.[19] The regressions of the excess returns on ISKEW and

[18]In Table IA.12 of the Supplementary Material, we report the results based on Fama–MacBeth regressions. We find the same conclusion.

[19]The results are similar with the sentiment index constructed using the partial least-squares method introduced by Huang, Jiang, Tu, and Zhou (2015).

the various controls are the same as before. The data range is from Aug. 1965 to Sept. 2015 due to the availability of the BW index data.

Table IA.13 of the Supplementary Material reports that conditional on high sentiment, skewness always has a significant negative effect on the expected return, regardless of whether or not there are other controls in place. However, when sentiment is low, its effects are always positive and significant. The sign change of the slopes confirms the earlier evidence that skewness cannot consistently explain excess returns.

In contrast, IE_{φ} and IS_{φ} have negative slopes regardless of the sentiment level, although their statistical significance is much stronger in high-sentiment periods. Although these results refer to excess returns, the results are qualitatively similar if the risk-adjusted returns are used. Overall, these results show that skewness is quite sensitive to sentiment, whereas IE_{φ} and IS_{φ} are much less so.[20]

D. Aggregate Stock Market Liquidity

Pástor, Stambaugh, and Taylor (2017) propose aggregate stock market liquidity (ALIQ) as another proxy for potential mispricing because mispricing is likely to be more prevalent when market liquidity is low. In this subsection, we examine how asymmetry effects vary during high- and low-ALIQ periods using Fama–MacBeth regressions. High-ALIQ periods are defined as those months when the realized ALIQ is above its sample mean. Likewise, low-ALIQ periods are defined as those months when the realized liquidity is below its sample mean.

We conduct regressions similar to equation (14), regressing excess returns on multiple measures of asymmetry and various controls for high-ALIQ and low-ALIQ periods separately. The results are shown in Table 8. The univariate regression results show a positive relation between ISKEW and the cross-section of future stock returns during high-ALIQ periods and a negative relation during low-ALIQ periods. The signs of these slopes may change when adding other controls.

In contrast, the relationship between IE_{φ} and expected returns is always significantly negative, regardless of the level of ALIQ. The same pattern is observed for IS_{φ}, although the negative coefficient is statistically insignificant for the univariate regression during high-ALIQ periods.

In summary, similar to our results for investor sentiment, the relationship between skewness and expected returns changes depending on the level of market liquidity. Theoretical models rarely consider evidence that skewness preference is state dependent. For example, instead of using historical skewness based on daily data, Conrad et al. (2013) use option price data to estimate skewness, and Amaya et al. (2015) use intraday data; they seem to find supporting evidence for unconditional models, although their samples are limited by the availability of data. In contrast, our new asymmetry measures do not suffer from these shortcomings and are consistent with theoretical models such as those of Barberis and Huang (2008), Goulding (2017), and Han et al. (2018), which predict that high upside asymmetry implies a lower expected return.

[20]An alternative explanation could be that current theoretical models miss the effect of sentiment on the correlation between skewness and the expected return.

TABLE 8

Fama–MacBeth Regressions in ALIQ Regimes

Table 8 reports the time-series average of the slope coefficients and their t-values (given in parentheses) from Fama–MacBeth regressions of stock excess returns on ISKEW, IE_{φ}, or IS_{φ}, as well as other variables representing firm characteristics (see column 1), from Aug. 1963 to Dec. 2015 in high- and low-ALIQ (aggregate stock market liquidity) periods. High-ALIQ periods are defined as those months when the realized ALIQ is above its sample mean. Likewise, low-ALIQ periods are defined as those months when the realized liquidity is below its sample mean.

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t} IA_{i,t} + \Lambda_t X_{i,t} + \varepsilon_{i,t+1}, \quad (15)$$

where $R_{i,t+1}$ is the excess return of stock i at time $t+1$; $IA_{i,t}$ is ISKEW, IE_{φ}, or IS_{φ} at time t for stock i; and $X_{i,t}$ is a set of control variables. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	High						Low					
	1	2	3	4	5	6	7	8	9	10	11	12
ISKEW	0.071**	−0.015					−0.068	−0.010				
	(2.14)	(−0.77)					(−1.64)	(−0.39)				
IE_{φ}			−3.403**	−4.495***					−4.502**	−3.655***		
			(−2.24)	(−4.94)					(−2.07)	(−3.04)		
IS_{φ}					−0.557	−0.998***					−1.284**	−0.958***
					(−1.54)	(−3.89)					(−2.39)	(−2.94)
SIZE		−0.219***		−0.220***		−0.221***		−0.178***		−0.175***		−0.178***
		(−5.55)		(−5.53)		(−5.56)		(−3.15)		(−3.09)		(−3.13)
BM		0.212***		0.209***		0.210***		0.322***		0.322***		0.320***
		(3.97)		(3.90)		(3.90)		(4.13)		(4.13)		(4.10)
MOM		0.011***		0.011***		0.011***		0.006**		0.006**		0.006***
		(7.89)		(8.01)		(8.02)		(2.56)		(2.58)		(2.60)
TURN		−0.115***		−0.109***		−0.109***		0.076		0.082		0.082
		(−3.13)		(−2.95)		(−2.95)		(1.50)		(1.63)		(1.63)
ILLIQ		0.063***		0.061***		0.062***		−0.023		−0.022		−0.021
		(3.76)		(3.67)		(3.69)		(−1.26)		(−1.20)		(−1.18)
β		0.990***		0.990***		0.993***		0.568*		0.562*		0.571*
		(4.87)		(4.88)		(4.89)		(1.85)		(1.83)		(1.86)
MAX		−0.014		−0.018		−0.019*		−0.071***		−0.073***		−0.075***
		(−1.22)		(−1.63)		(−1.75)		(−4.89)		(−5.08)		(−5.26)
IVOL		−0.408***		−0.397***		−0.399***		−0.248***		−0.240***		−0.240***
		(−9.81)		(−9.58)		(−9.65)		(−4.77)		(−4.69)		(−4.67)
Constant	0.773***	2.124***	0.821***	2.122***	0.814***	2.132***	0.466	1.922***	0.449	1.891***	0.447	1.908***
	(3.35)	(7.94)	(3.52)	(7.90)	(3.48)	(7.95)	(1.25)	(4.66)	(1.19)	(4.60)	(1.19)	(4.64)
R^2	0.003	0.075	0.001	0.075	0.001	0.075	0.003	0.099	0.002	0.098	0.001	0.098

E. Capital Gains Overhang

In this subsection, we examine how the effect of asymmetry on stock returns varies with the capital gains overhang (CGO) using different measures. An, Wang, Wang, and Yu (2018) find that the negative relation between skewness and expected return depends on the CGO level and is confined to stocks that experience capital losses. It is of interest to investigate whether our new asymmetry measures are subject to the same constraints.

To examine the effect of CGO on the relationship between asymmetry and expected returns, we conduct a double-sort analysis. At the beginning of each month from Aug. 1963 to Dec. 2015, we first sort stocks by CGO into quintile portfolios. Then, within each CGO portfolio, we sort stocks into quintile portfolios by one of the following asymmetry measures: ISKEW, IE_{φ}, or IS_{φ}. For brevity, Table 9 reports the results only for the highest- and lowest-asymmetry portfolios. When using skewness to measure asymmetry, we see a negative and statistically significant return only on the spread portfolio of P5 − P1 (the difference between the highest- and lowest-skewness stocks) in the lowest quintile of CGO, which reaffirms that skewness is tied to the CGO level. In contrast, the spread portfolios for IE_{φ} and IS_{φ} have mostly significant returns across the

TABLE 9

Portfolios Sorted by CGO and Asymmetry Measures

Table 9 reports the average returns and their t-values for quintile portfolios sorted by capital gains overhang (CGO) and then by ISKEW, IE_{φ}, or IS_{φ} from Aug. 1963 to Dec. 2015. CGO1 and CGO5 denote the lowest and highest quintiles for CGO, respectively. P1 and P5 denote the lowest and highest quintiles for ISKEW, IE_{φ}, and IS_{φ}, respectively. t-statistics are given in parentheses. ** and *** indicate significance at the 5% and 1% levels, respectively.

	ISKEW			IE_{φ}			IS_{φ}		
Proxy	P1	P5	P5 – P1	P1	P5	P5 – P1	P1	P5	P5 – P1
CGO1	0.713***	0.199	−0.514***	0.576**	0.380	−0.195**	0.548**	0.334	−0.214***
	(2.82)	(0.77)	(−4.68)	(2.45)	(1.49)	(2.15)	(2.24)	(1.31)	(−2.59)
CGO2	0.563***	0.414*	−0.149	0.627***	0.447**	−0.179**	0.679***	0.429*	−0.250***
	(2.65)	(1.76)	(−1.54)	(2.94)	(1.98)	(−2.52)	(3.07)	(1.86)	(−3.74)
CGO3	0.511***	0.608***	0.097	0.677***	0.589***	−0.089	0.703***	0.612***	−0.090
	(2.75)	(2.75)	(1.00)	(3.51)	(2.79)	(−1.26)	(3.47)	(2.80)	(−1.32)
CGO4	0.665***	0.754***	0.090	0.816***	0.652***	−0.164**	0.846***	0.759***	−0.087
	(3.67)	(3.62)	(1.02)	(4.46)	(3.18)	(−2.37)	(4.34)	(3.63)	(−1.34)
CGO5	0.929***	1.088***	0.158*	1.215***	1.128***	−0.088	1.244***	1.152***	−0.092
	(4.97)	(5.25)	(1.71)	(6.34)	(5.28)	(−1.20)	(6.14)	(5.13)	(−1.24)
Avg(C1 – C5)	0.676***	0.613***	−0.064	0.782***	0.639***	−0.143***	0.804***	0.657***	−0.147***
	(3.48)	(2.81)	(−0.87)	(4.03)	(3.01)	(−3.02)	(3.95)	(3.02)	(−3.63)

CGO quintiles. Therefore, whereas the effect of skewness is closely related to CGO, our new measures of asymmetry are fairly robust to variation in CGO.

VI. Conclusion

In this article, we propose two distribution-based measures of stock return asymmetry. The first measure is based on the probability difference between the upside potential and downside loss of a stock, and the second measure is an entropy-scaled version of the first. Our measures incorporate more information (i.e., the density function of the data) than the widely used skewness measure, which relies on the third moment only. As a result, they capture asymmetry more effectively, as shown in simulations and empirical results.

Based on our new measures, we find that greater asymmetries imply lower average returns in the cross section of stock returns. This association is statistically significant not only at the firm level but also in the cross section of portfolios sorted by the new asymmetry measures. The results are robust to a number of controls, such as volatility, investor sentiment, market liquidity, and the capital gains overhang. In contrast, the corresponding empirical results using skewness are inconclusive. Hence, our proposed measures are valuable complements to skewness.

Our empirical results are consistent with the predictions of theoretical models, such as Han et al. (2018). For future research, it would be of interest to apply our measures to other asset classes, such as bonds and foreign exchange, as well as to portfolio choice problems.

Appendix A. The Proof

In Appendix A, we show analytically that under certain simplifying assumptions and everything else equal, greater IE_{φ} is associated with lower expected returns.

Consider a simple representative investor economy with only one risky asset. We denote the 1-period return of the asset by r. There is also a risk-free asset with return r_f.

We assume the representative investor lives for only 2 periods, period 0 and 1, and has a utility function U, to be specified later. Let W_0 denote the initial wealth in period 0 and W_1 denote the final wealth in period 1. Then the investor's first-order condition or the Euler equation is

$$\text{(A-1)} \qquad E[U'(W_1)(r-r_f)] = 0,$$

with

$$\text{(A-2)} \qquad W_1 = W_0\cdot(wr+(1-w)r_f),$$

where w is the optimal portfolio weight. Equivalently, we have

$$\text{(A-3)} \qquad \begin{aligned} E[U'(W_1)]E[r-r_f] &= -\text{Cov}(U'(W_1),r-r_f) \\ &= -\text{Cov}(U'(W_1),r). \end{aligned}$$

Denote by μ and σ^2 the mean and variance of r, and then

$$\text{(A-4)} \qquad \text{Cov}(U'(W_1),r) = \sigma\cdot\text{Cov}\left(U'(W_1),\frac{r-\mu}{\sigma}\right) = \sigma\cdot\text{Cov}(U'(W_1),\widetilde{r}),$$

where $\widetilde{r}$ is the standardized return r. Denote by $f(\widetilde{r})$ its probability density function, and we have

$$\text{(A-5)} \qquad \begin{aligned} \text{Cov}(U'(W_1),r) &= \sigma\cdot\text{Cov}(U'(W_1),\widetilde{r}) \\ &= \sigma\cdot E[U'(W_1)\widetilde{r}]-\sigma\cdot E[U'(W_1)]\cdot E[\widetilde{r}] \\ &= \sigma\cdot E[U'(W_1)\widetilde{r}] \\ &= \sigma\int_{-\infty}^{+\infty}U'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r}. \end{aligned}$$

Now we want to link the previous equations to IE_φ. Without loss of generality, we assume $r_f=0$ and $W_0=1$. We define the utility function as follows:

$$\text{(A-6)} \qquad U(W_1) = \begin{cases} \ln(W_1-\mu), & \text{if } W_1-\mu \geq \sigma; \\ u(W_1), & \text{if } \sigma > W_1-\mu > -\sigma, \\ -\ln(\mu-W_1), & \text{if } W_1-\mu \leq -\sigma, \end{cases}$$

where we assume $\sigma\geq 1$ and $u(W_1)$ is a utility function that is smooth at the boundaries.

In equilibrium, we have, based on the market-clearing conditions requiring $w=1$,

$$\text{(A-7)} \qquad \begin{aligned} E[U'(W_1)]E[r] &= -\sigma\int_{-\infty}^{-1}U'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r} \\ &\quad -\sigma\int_{1}^{+\infty}U'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r}-\sigma\int_{-1}^{1}U'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r} \\ &= -\text{IE}_\varphi-\sigma\int_{-1}^{1}u'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r}. \end{aligned}$$

Denote $\xi=-1/E[U'(W_1)]$. It is clear that $\xi<0$. The previous equation implies that

$$\text{(A-8)} \qquad E[r] = \xi\text{IE}_\varphi-\sigma\xi\int_{-1}^{1}u'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r}.$$

Hence, everything else equal, a greater value of IE_φ implies a lower expected return.

It is interesting to note that there is an extra term, $-\sigma\xi\int_{-1}^{1}u'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r}$, that also affects the expected return. The questions are what the impact is and whether controlling it drives out the IE_φ effect. To address these questions, we consider two specifications of the utility function:

$$\text{(A-9)} \qquad u_1(W_1) = \frac{\ln\sigma}{\sigma}(W_1-\mu), \quad \text{if } \sigma > W_1-\mu > -\sigma$$

and

$$\text{(A-10)} \qquad u_2(W_1) = \frac{\ln\sigma}{\sigma^{1/3}}(W_1-\mu)^{1/3}, \quad \text{if } \sigma > W_1-\mu > -\sigma.$$

The first is a typical utility function of risk-neutral investors, and the second is often used to represent concave in capital gain and convex in capital loss. Then the effects of the extra term can be summarized by U_j^I, defined from

$$\text{(A-11)} \qquad -\sigma\xi\int_{-1}^{1}u_j'(W_1)\widetilde{r}f(\widetilde{r})d\widetilde{r} = -\xi U_j^I,$$

where $j=1$ and 2. Theoretically, both U_1^I and U_2^I are positively related to the expected returns. Consistent with this, Tables IA.14 and IA.15 of the Supplementary Material show empirically that a greater U_1^I or U_2^I implies a higher expected return. Moreover, after controlling for U_1^I and U_2^I, IE_φ or IS_φ still has a significant impact on the expected return.

Appendix B. Variable Definitions

In Appendix B, we provide detailed definitions of all of the variables used in the article.

R: The excess return of stock i in month t is calculated using the difference between the monthly stock return on stock i and the 1-month T-bill rate at time $t+1$.

RA: Following Brennan, Chordia, and Subrahmanyam (1998), the risk-adjusted return of stock i in month $t+1$ is defined as the excess return adjusted for the Fama–French 3 factors at time $t+1$.

IE_φ: The excess tail probability of stock i in month t evaluated at 1 standard deviation away from the mean. IE_φ is defined in equation (1). x is the standardized residual after adjusting for the market return. Following Harvey and Siddique (2000), when estimating idiosyncratic measurements other than volatility, we utilize the daily residuals $\varepsilon_{i,d}$ in the following equation:

$$\text{(B-1)} \qquad R_{i,d} = \alpha_i + \beta_i \cdot R_{m,d} + \gamma_i \cdot R_{m,d}^2 + \varepsilon_{i,d},$$

where $R_{i,d}$ is the excess return of stock i on day d, $R_{m,d}$ is the market excess return on day d, and $\varepsilon_{i,d}$ is the idiosyncratic return on day d. We use daily residuals $\varepsilon_{i,d}$ from month t to $t-11$ to calculate IE_φ.

IS_φ: The scaled version of the excess tail probability of stock i in month t evaluated at 1 standard deviation away from the mean. IS_φ is defined in equation (4). x is the standardized residual after adjusting for the market return. Similar to IE_φ, we use daily residuals $\varepsilon_{i,d}$ in equation (B-1) from month t to $t-11$ to calculate IS_φ.

VOL: The volatility of stock i in month t, defined as the standard deviation of daily returns within month t:

$$\text{(B-2)} \qquad \text{VOL}_{i,t} = \sqrt{\text{var}(R_{i,d})}, \quad d = 1,\ldots,D_t,$$

where D_t is the number of trading days for month t.

IVOL: The idiosyncratic volatility of stock i in month t, defined as the standard deviation of daily idiosyncratic returns within month t. To calculate return residuals, we adjust for the market return:

$$R_{i,d} = \alpha_i + \beta_i \cdot R_{m,d} + \varepsilon_{i,d}, \quad d = 1,\ldots,D_t, \tag{B-3}$$

where $\varepsilon_{i,d}$ is the idiosyncratic return on day d for stock i, and D_t is the number of trading days for month t. The IVOL of stock i in month t is then defined as follows:

$$\text{IVOL}_{i,t} = \sqrt{\text{var}(\varepsilon_{i,d})}, \quad d = 1,\ldots,D_t. \tag{B-4}$$

SKEW: The skewness of stock i in month t, computed using daily returns from month t to $t-11$:

$$\text{SKEW}_{i,t} = \frac{1}{D_t}\sum_{d=1}^{D_t}\left(\frac{R_{i,d}-\mu_i}{\sigma_i}\right)^3, \tag{B-5}$$

where D_t is the number of trading days in a year, $R_{i,d}$ is the excess return on stock i on day d, μ_i is the mean of the returns of stock i over a year, and σ_i is the standard deviation of returns of stock i over a year.

ISKEW: The idiosyncratic skewness of stock i in month t, computed using the daily residuals $\varepsilon_{i,d}$ in equation (B-1) from month t to $t-11$.

ISK_α or ISK_{INT}: The alternative idiosyncratic skewness measures that follow Ghysels et al.'s (2016) definition. ISK_α of stock i in month t is computed using daily residuals $\varepsilon_{i,d}$ in equation (B-1) from month t to $t-11$:

$$\text{ISK}_{\alpha,i,t} = 6\cdot\frac{q_{\alpha,i,t}+q_{1-\alpha,i,t}-2\cdot q_{0.50,i,t}}{(q_{\alpha,i,t}-q_{1-\alpha,i,t})\cdot q_\alpha(z)}, \tag{B-6}$$

$$\text{ISK}_{\text{INT},i,t} = 6\cdot\frac{\int_{0.5}^{1}[q_{\alpha,i,t}+q_{1-\alpha,i,t}-2\cdot q_{0.50,i,t}]\,d\alpha}{\int_{0.5}^{1}[q_{\alpha,i,t}-q_{1-\alpha,i,t}]\,d\alpha}\cdot\frac{\int_{0.5}^{1}q_\alpha(z)\,d\alpha}{\int_{0.5}^{1}q_\alpha^2(z)\,d\alpha}, \tag{B-7}$$

where $q_{\alpha,i,t}$ is the α conditional quantile of daily residuals of stock i in a year, and $q_\alpha(z)$ is the α quantile of a standard Gaussian random variable z. We apply open Newton–Cotes formulae, use the Gillenator method (Press, Teukolsky, Vetterling, and Flannery (2007)), and choose points at $\{0.6, 0.7, 0.8, 0.9\}$ to calculate the integration.

β: The market beta, calculated as

$$R_{i,d} = \alpha + \beta_{i,y}\cdot R_{m,d} + \varepsilon_{i,d}, \quad d = 1,\ldots,D_y, \tag{B-8}$$

where $R_{i,d}$ is the excess return of stock i on day d, $R_{m,d}$ is the market excess return on day d, and D_y is the number of trading days in year y. β is annually updated.

COSKEW: Following Harvey and Siddique (2000) and Chabi-Yo (2012), the coskewness of stock i over a 12-month period ending in month t, defined as

$$\text{COSKEW}_{i,t} = \frac{\frac{1}{D_t}\sum_{d=1}^{D_t}R_{i,d}R_{m,d}^2}{\sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t}R_{i,d}^2}\left(\frac{1}{D_t}\sum_{d=1}^{D_t}R_{m,d}^2\right)}, \tag{B-9}$$

where D_t is the number of trading days in a 12-month period ending in month t, $R_{i,d}$ is the excess return on stock i on day d, and $R_{m,d}$ is the market excess return on day d.

COKURT: Following Dittmar (2002) and Chabi-Yo (2012), the cokurtosis of stock i over a 12-month period ending in month t, given by

$$\text{(B-10)} \qquad \text{COKURT}_{i,t} = \frac{\frac{1}{D_t}\sum_{d=1}^{D_t} R_{i,d} R_{m,d}^3}{\sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t} R_{i,d}^2}\left(\frac{1}{D_t}\sum_{d=1}^{D_t} R_{m,d}^2\right)^{3/2}},$$

where D_t is the number of trading days in a 12-month period ending in month t, $R_{i,d}$ is the excess return on stock i on day d, and $R_{m,d}$ is the market excess return on day d.

MAX: The maximum daily return (in percentage points) in a given month following Bali et al. (2011):

$$\text{(B-11)} \qquad \text{MAX}_{i,t} = \text{MAX}(R_{i,d}), \quad d = 1,\ldots,D_t,$$

where $R_{i,d}$ is the excess return of stock i on day d, and D_t is the number of trading days in month t.

MIN: The negative of the minimum daily return (in percentage points) in a given month following Bali et al. (2011):

$$\text{(B-12)} \qquad \text{MIN}_{i,t} = -\text{MIN}(R_{i,d}), \quad d = 1,\ldots,D_t,$$

where $R_{i,d}$ is the excess return of stock i on day d, and D_t is the number of trading days in month t.

PRC: The price at the end of month t.

SIZE: Firm size (in $millions) at each month t, measured using the natural logarithm of the market value of equity at the end of month t.

BM: Book-to-market ratio; following Fama and French (1992), (1993), a firm's BM is calculated using the market value of equity at the end of December of the prior year and the book value of common equity plus the balance-sheet deferred taxes for the firm's fiscal year ending in the prior calendar year. We assume book value is available 6 months after the reporting date. Our measure of BM at month t is defined as the natural logarithm of the book-to-market ratio at the end of month t.

MOM: Following Jegadeesh and Titman (1993), the momentum effect (in percentage points) of each stock in month t, measured by the cumulative return over the previous 6 months with 1 month skipped (i.e., the cumulative return from month $t-6$ to month $t-1$).

REV: Following Jegadeesh (1990) and Lehmann (1990), the short-term reversal (in percentage points) for each stock in month t, defined as the excess return on the stock over the previous month (i.e., the return in month $t-1$).

REVA or REVA4: The adjusted short-term reversal (in percentage points), defined as the excess return adjusted either for the Fama–French 3 factors or the Fama–French 3 factors and the tail-risk factor (see Brennan et al. (1998) and Kelly and Jiang (2014)) over the previous month.

TURN: The turnover (in percentage points), calculated as the monthly trading volume divided by the outstanding month-end shares.

ILLIQ: Illiquidity; following Amihud (2002), we first calculate the ratio of the absolute price change to the dollar trading volume for each stock on each day. Then we take the average of the ratio for the month if the number of observations is higher than 15. Following Acharya and Pedersen (2005), we normalize the Amihud ratio and truncate it at 30.

O_SCORE and P_CHS: Measures of financial distress (in percentage points). Following Ohlson (1980), O_SCORE is a weighted combination of financial ratios with coefficients estimated from a dynamic logit model to predict bankruptcy. Following Campbell et al. (2008), (2011), we first construct the financial distress measure CHS by using 3 accounting-based variables and 5 market-based variables. P_CHS is the probability of default then. The higher the O_SCORE and P_CHS, the higher is the risk of default.

CGO: The capital gains overhang at week w, defined as

$$\text{CGO}_w = \frac{P_{w-1} - \text{RP}_w}{P_{w-1}}, \tag{B-13}$$

where P_{w-1} is the stock price at the end of week $w-1$, and RP_w is the reference price for each individual stock. As in Grinblatt and Han (2005), we use information for the past 260 weeks (with at least 200 valid price and turnover observations) to calculate the reference price for each stock. RP_w is defined as follows:

$$\text{RP}_w = k^{-1} \sum_{n=1}^{260} \left(V_{w-n} \prod_{\tau=1}^{n-1} (1 - V_{w-n+\tau}) \right) P_{w-n}, \tag{B-14}$$

where V_w is the turnover in week w, and k is the constant that makes the weights on past prices sum to 1.

Supplementary Material

Supplementary Material for this article is available at https://doi.org/10.1017/S0022109019000206.

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